Design of Nested LDGM-LDPC Codes for Compress-and-Forward in Relay Channel

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***Abstract*—A three terminal relay system with binary erasure channel (BEC) was considered, in which a source forwarded information to a destination with a relay's "assistance". The nested LDGM (Low-density generator-matrix) -LDPC (low-density parity-check) was designed to realize Compress-and-forward (CF) at the relay. LDGM coding compressed the received signals losslessly and LDPC realized the binning for Slepian-Wolf coding. Firstly a practical coding scheme was proposed to achieve the cut-set bound on the capacity of the system, employing LDPC and Nested LDGM-LDPC codes at the source and relay respectively. Then, the degree distribution of LDGM and LDPC codes was optimized with a given rate bound, which ensured that the iterative belief propagation (BP) decoding algorithm at the destination was convergent. Finally, simulations results show that the performance achieved based on nested codes is very close to Slepian-Wolf theoretical limit.**

***Index Terms*—Slepian-Wolf source coding, Nested LDGM-LDPC, Compress-and-Forward, BEC, Relay channel.**

I. Introduction

COOPERATIVE communication [1] has recently attracted much attention due to it can achieve a larger rate region, compared with traditional networks. Many cooperative protocols have been proposed in the literature. These protocols are usually classified into three categories: decode-and-forward (DF) protocol, where the relay decodes and re-encodes the signals transmitted by the source; amplify-and-forward (AF) protocol, in which a relay simply amplifies its received signals; compress-and-forward (CF) protocol, where the relay compresses the signals from the source, and forwards these compressed soft information to the destination. For DF protocol, it has been widely researched based on LDPC codes [2], and suffers a loss of performance when the relay can't be guaranteed to recover the source information. The relay always can assist the source to convey information with amplified soft information when AF protocol is employed, but the protocol is suboptimal. CF protocol, jointing source-channel coding, is a form of Wyner-Ziv (WZ) coding [3] in case of lossy compression and Slepian-Wolf coding [4] in the lossless case. It takes advantages of the statistical dependence of the relay's and destination's channel output, and achieves higher rate than DF and AF.

So far, most of the researches concerning CF remain at the theoretical level and the realization of CF is tackled in only a few papers, [5] [6] [7]. These papers achieve Slepian-Wolf compression at the relay by taking the syndrome of an LDPC code [5], or by using an Irregular Repeat Accumulator (IRA) code [6] that combines Slepian-Wolf coding with channel coding on the relay-to-destination channel. However these approaches are suitable only when the source-relay channel output is binary and the relay does lossless (rather than lossy) compression. Otherwise, e.g. the Gaussian relay channel is considered in [7], significant capacity losses will result.

It is known that LDPC codes are good channel codes and recent work has also shown that LDGM codes are good source codes. Some near-ideal encoding/decoding algorithms with LDGM codes have been proposed [8] [9], furthermore nested LDGM-LDPC codes often are used to guarantee both channel coding and source coding performance. In [10], such nested codes are used to approach capacity in dirty paper coding, where good channel coding ensures low error probability and good source coding guarantees good shaping of the transmitted signal.

To focus on the effective compression of the received data at the relay, we consider a three terminal cooperative system where the source-destination and source-relay links are both binary erasure channel (BEC), and the relay-destination link is orthogonal to them. As the received signal by the relay through the BEC is 3-ary, the aforementioned syndrome methods are insufficient, making the proposed nested codes necessary.

This paper is organized as follows. Section II gives a brief introduction to the system model involving a relay channel. Section III describes the compression and decoding algorithms based on nested LDGM-LDPC codes. Section IV presents the degree distribution optimization method necessary for good performance. Experiment results are given in Section V to verify the effectiveness of the proposed algorithms and optimization methods. Finally, Section VI concludes the paper.

II. System Model

The full-duplex single relay system is shown in Fig.1. It comprises a source S, a destination D and a relay node R. The S-R and S-D links, both with erasure probability ε, form a binary erasure broadcast channel. The R-D link, with a capacity denoted by C_{rd}, is orthogonal to the S-D and S-R links. Denoting an erased bit by E, the relay system is described by four random variables x_s, x_r, y_r, y_d and

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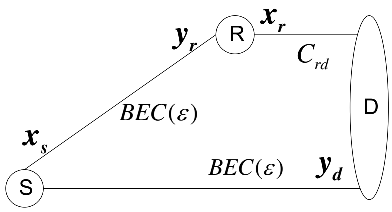

Fig. 1. The single relay system

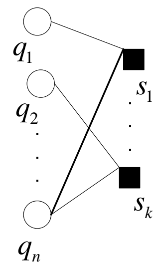

Fig. 2. The LDPC code C_1

one conditional probability distribution $p(y_r, y_d|x_s)$,which is shown in Table I.

TABLE I
CONDITIONAL PROBABILITY DISTRIBUTION OF $p(y_r, y_d|x_s = 0)$, WHEN $p(y_r, y_d|x_s = 1)$, THE VARIABLES VALUE: 0 AND 1 EXCHANGE

y_d, y_r	0	1	E
0	$(1-\varepsilon)^2$	0	$(1-\varepsilon)\varepsilon$
1	0	0	0
E	$(1-\varepsilon)\varepsilon$	0	ε^2

In every block a message w which is random variable uniformly distributed on $[1, 2^{nR})$ is encoded into sequence $\boldsymbol{x_s} = (x_{s1}, x_{s2}, \cdots, x_{sn})$ at the source S, and transmitted through BEC. $\boldsymbol{y_r} = (y_{r1}, y_{r2}, \cdots, y_{rn})$, $\boldsymbol{y_d} = (y_{d1}, y_{d2}, \cdots, y_{dn})$ is received by R and D respectively. At R $\boldsymbol{y_r}$ is decoded or loselessly compressed, and then re-encoded into $\boldsymbol{x_r} = (x_{r1}, x_{r2}, \cdots, x_{rn'})$ to implement DF or CF respectively. When C_{rd} is sufficiently large, $\boldsymbol{y_r}$ could be decoded with the compressed signals $\boldsymbol{x_r}$ and the side information $\boldsymbol{y_d}$. w could be recovered at D by jointly decoding $\boldsymbol{y_r}$ and $\boldsymbol{y_d}$. Then the achievable rate [11] of the system is given by

1) If $\boldsymbol{y_r}$ is decoded, then

$$R \leq I(x_s; y_r|x_r) = 1 - \varepsilon \tag{1}$$

2) If $\boldsymbol{y_r}$ is compressed and encoded into $\boldsymbol{x_r}$, with large C_{rd} the compression can be lossless, in which case

$$R \leq I(x_s; y_r y_d) = 1 - \varepsilon^2 \tag{2}$$

It can be seen from (1) that, when C_{rd} is large, the S-R link become the bottleneck of DF mode. However CF can achieve a higher rate in (2), which is actually the cut-set bound [11] on the capacity of the relay channel. On the other hand, in CF mode the relay node is unable to decode $\boldsymbol{y_r}$, so the C_{rd} must be large enough to transmit all the information about the erased positions.

In this paper, we focus on lossless CF, and since it already achieves capacity, the remaining task is to minimize C_{rd} the necessary. As $\boldsymbol{y_d}$ is available at D and is correlated with $\boldsymbol{y_r}$, it can be used as side information to reduce $\boldsymbol{y_r}$'s encoding rate from $H(y_r)$ into $H(y_r|y_d)$ through Slepian-Wolf coding. In the next section, nested LDGM-LDPC codes will be designed to realize the compression and binning necessary for Slepian-Wolf coding.

III. THE NESTED LDGM-LDPC CODE FOR COMPRESSION

Considering that LDPC as channel code achieves capacity-approaching performance with low-complexity iterative decoding manner, we encode the source messages with an LDPC code, denoted by factor graph [12] C_1, which is shown in Fig.2. The circles denote the variable nodes q, representing n bits codeword of LDPC codes, and the black squares denote the check (function) nodes s, representing k parity check equations. The number of edges connected to one node is denoted as the degree of the node. The rate $R_0 = \frac{n-k}{n}$ could approach cut-set bound in (2) by optimizing the degree distribution of C_1, which will is discussed in the next section.

With C_1 the message w at S is encoded into a binary sequence $\boldsymbol{x_s} \in \{0,1\}^n$ and transmitted over BEC. Then the received sequence $\boldsymbol{y_r}$ by R is relayed to D via CF. We design a nested LDGM-LDPC construction C_2, to deal with the compression problem at R, whose factor graph is shown in Fig.3. The circles are also the variable nodes and black squares are the factor nodes. As each is regarded as a ternary symbol, we firstly map $\boldsymbol{y_r}$ into a binary sequence $\boldsymbol{c}$ by

$$\begin{aligned} \hat{y}_{ri} &= \varphi(c_{2i-1} \oplus v_{2i-1} \oplus \zeta_{2i-1}, c_{2i} \oplus v_{2i} \oplus \zeta_{2i}), \\ \varphi(00) &= 0, \varphi(01) = 1, \varphi(1*) = E, i = 1, 2, \cdots, n, \end{aligned} \tag{3}$$

Where $*$ denotes "don't care" positions that can be encoded into either 0 or 1. $\boldsymbol{v}$ is a pseudo-random dither sequence which is added to assure uniform distribution. $\boldsymbol{\zeta}$ is normally an all-zero sequence, but in practice erroneous decimation of the b-nodes will inevitably occur and cause contradictions, which must be corrected by flipping the bits in corresponding to the c-nodes with contradictions. Here the factor nodes connected to $\boldsymbol{y_r}$ and $\boldsymbol{c}$ represent the mapping in (3). Then with LDGM part of C_2 , the binary sequence containing $*$ is quantized into a shorter one $\boldsymbol{b}$ by

$$c = bG, b \in \{0,1\}^m, m = nR_b \tag{4}$$

where G is the generation matrix and R_b is optimized to be slightly larger than $2 - \varepsilon$, so that LDGM coding with R_b is lossless. After that, we use the LDPC part of G_2 to compress $\boldsymbol{b}$ into $\boldsymbol{p}$, with

$$p = bH^T, p \in \{0,1\}^t, t = nR_p \tag{5}$$

where H is the sparse parity check matrix and R_p also optimized is slightly larger than $H(y_r|y_d)$. $H(y_r|y_d)$ is the Slepian-Wolf theoretical limit.

Until now the nested LDGM-LDPC code has been used to compress $\boldsymbol{y_r}$ into $\boldsymbol{p}$ (as well as the flipped positions $\boldsymbol{\zeta}$) at the rate approximately equal to $H(y_r|y_d)$. Assuming that C_{rd} is sufficient for transmitting these information, D can decode $\boldsymbol{y_r}$ from the side information $\boldsymbol{y_r}$ and the compressed information

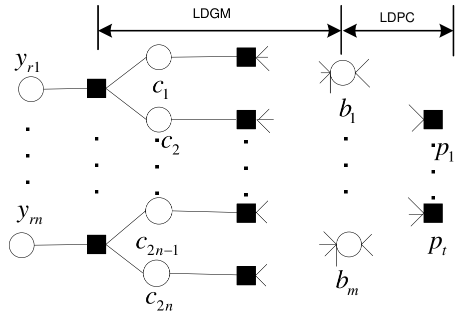

Fig. 3. The nested LDGM-LDPC codes C_2

$\boldsymbol{p}$ losslessly. The iterative belief propagation [13] decoding process is also executed from Fig.3. Finally, combining $\boldsymbol{y_r}$ with $\boldsymbol{y_d}$, D recovers the message w, again by the BP algorithm.

IV. Code Optimization C_1 and C_2

When LDPC, LDGM and nested codes are designed, it is always critical to optimize the degree distributions so that BP converges well, which can be visualized on the EXIT chart [14] as a gap. So we optimize the degree distribution using the EXIT and EBP curve (Extended BP curve which is another form of EXIT chart) [15] considering both the encoding process and decoding process also from the source node, relay node and destination node.

The design of C_1 involves just LDPC optimization over a BEC with erasure probability ε^2 , which means that the prior information at q-nodes acquired from channel output becomes $I_{q,pri} = 1 - \varepsilon^2$. S-regular, q-irregular LDPC code [13] is designed to achieve good performance. Let d_s be the left-degree of all s-nodes, and denote v_{qd} as the fraction of edges connected to q-nodes with the right-degree d. I_{qs}, I_{sq} represent the average mutual information (MI) in every q-to-s, s-to-q message at a certain iteration respectively. Then the optimization the degree distribution v_{qd} of q-nodes is a linear programming problem, which is

$$\begin{aligned} &maxR_0 = 1 - \frac{1/d_s}{\sum_d v_{qd}/d} \\ &s.t. \sum_d v_{qd} = 1, I_{qs} > I_{qs}^{-} + \triangle_{qs}, I_{sq} \in [0,1] \end{aligned} \tag{6}$$

in which

$$I_{qs} = 1 - (1 - I_{q,pri}) \sum_d v_{qd}(1 - I_{sq})^{d-1} \tag{7}$$

$$I_{qs}^{-} = (I_{sq})^{\frac{1}{d_s - 1}} \tag{8}$$

where superscript ”-” refers to last iteration. EXIT curves at q-nodes and s-nodes called q-curve and s-curve are given by (7) and (8). $\triangle_{qs}$ is added to ensure that there are some gap between the matched EXIT curves, so that BP algorithm won’t get stuck.

For the nested LDGM-LDPC code C_2, the LDGM part is essentially dictated by good encoder-side performance at the relay node, so we optimize it first. As Ternary symbol y_{ri} is encoded into two bits c_{2i-1},c_{2i} in c-nodes, to simplify analysis, we assume all b-nodes have the left-degree d_b and the two bits c_{2i-1},c_{2i} connected to the same y_{ri}-node have the same right-degree, called the c-degree of the y_r-node. Now we only have to optimize the c-degree distribution of the y_r-nodes, represented by v_{cd}, the fraction of edges connected to y_r-nodes with c-degree d from the edge perspective. Besides, the optimization is needed with the constraint of the monotonic condition [9], which makes sure that encoding can proceed with a vanishing fraction of contradictions and thus flipped bits. Thus the optimization problem is summarized as

$$\begin{aligned} &\min_{v_{cd}} R_b = \frac{2}{d_b \sum_d v_{cd}/d} \\ &s.t. \sum_d v_{cd} = 1, I_{bc,pri}|_{I_{bc}=0} \geq 0, \frac{dI_{bc,pri}}{dI_{bc}} \geq 0, I_{bc} \in [0,1] \end{aligned} \tag{9}$$

in which

$$I_{bc,pri} = 1 - (1 - I_{bc})/(1 - I_{cb})^{d_b - 1} \tag{10}$$

$$I_{cb} = I_{yc} \textstyle\sum_d v_{cd}(I_{bc})^{d-1} \tag{11}$$

Where I_{bc}, I_{cb} denotes the average MI in every b-to-c, c-to-b message at a certain iteration respectively, and $I_{yc} = 1 - 0.5\varepsilon$ represents the priors average MI of the y_r-nodes with c-degree, which is acquired from the mapping in (3). $I_{bc,pri}$ denotes the priors average MI of the b-nodes at fixed points(i.e. $I_{bc,pri}$ making the average MI $I_{bc} = I_{bc}^{-}$), and it should be 0 when $I_{bc} = 0$ and increase monotonically as I_{bc} increase from 0 to 1.

With the degree distribution of LDGM part fixed, we optimize the degree distribution of the LDPC part of C_2 to achieve decoder-side performance. During the iterative decoding of $\boldsymbol{y_r}$ at the destination, the average MI of the message from y_r- to c-nodes, denoted as $I_{yc,d}$ for those with c-degree d, varies between $I_{c0} = I(c; y_d)$ and $I_{c1} = I(c_{2i-1}; y_{di}|c_{2i}) + I(c_{2i}; y_{di}|c_{2i-1})$, according to the incoming messages from c- and -nodes, that is

$$I_{yc,d} = I_{c0}(1 - I_{bc}^d) + I_{c1} I_{bc}^d \tag{12}$$

Making I_{cb} in (11) become

$$I_{cb} = \sum_d v_{cd} I_{yc,d} (I_{bc})^{d-1} \tag{13}$$

Let $I_{bc,ext}$ denote the extrinsic MI of b-node at fixed points, derived only from I_{cb}, which is

$$I_{bc,ext} = 1 - (1 - I_{cb})^{d_b} \tag{14}$$

Thus the decoder-side EBP curve of the LDGM part formed by $I_{bc,pri}$ vs. $I_{bc,ext}$ is derived from (10) and (14). The LDPC part is designed to make the EBP curve of LDPC part $I_{bp,pri}$ vs. $I_{bp,ext}$ match that of LDGM part, so that $\boldsymbol{y_r}$ can be decoded. In other words, suppose the EBP curve is plotted with $I_{bp,ext}$ in the horizontal axis and $I_{bc,ext}$ in the vertical axis, then EBP curve of the LDPC part should lie below that of the LDGM part, with a small gap between them. The gap assures that iterative decoding does not get stuck. Thus let v_{pd} and v_{bd}

denotes the fraction of edges connected to p-node and b-node with the left-degree and right-degree d respectively. The degree distributions are optimized to achieve the minimal rate, which is

$$\begin{aligned}
&\min_{v_{pd},v_{bd}} R_p = R_b - R_{bp} = R_b - (1 - \frac{\sum_d v_{pd}/d}{\sum_d v_{bd}/d}) \\
&s.t. \sum_d v_{pd} = 1, \sum_d v_{bd} = 1, I_{pb}^{+} > I_{pb} + \triangle_{pb}, I_{pb} \in [0,1] \\
&I_{bp,ext} = 1 - \sum_d v_{bd}(1 - I_{pb})^d \\
&I_{bc,pri} = I_{bp,ext}, I_{bp,pri} = I_{bc,ext} \\
&I_{bp} = 1 - (1 - I_{bp,pri}) \sum_d v_{bd}(1 - I_{pb})^{d-1} \\
&I_{pb}^{+} = \sum_d v_{pd}(I_{bp})^{d-1}
\end{aligned} \tag{15}$$

Where $I_{bc,ext}$ is derived from the EBP curve of LDGM part at the decoder-side with the corresponding $I_{bc,pri}$ known. I_{bp}, I_{pb} denotes the average MI in every b-to-p, p-to-b message at a certain iteration respectively, and superscript "+" refers to the next iteration. $\triangle_{pb}$ is designed to keep the gap in the EBP curves of the LDGM and LDPC part to make the BP converge.

V. Numerical Results

In this section we evaluate the performance of our optimization for degree distribution and LDGM-LDPC encoding and decoding process. Let the erasure probabilities of the S-R and S-D links be $\varepsilon = 0.5$, the R-D link be ideal with C_{rd} at least 1.25 bit/sym, and the block length $n = 10^5$. Since the cut-set bound in (2) is $I(x_s; y_d y_d) = 0.75$ bit/sym, the code rate of C_1 is $R_0 \leq 0.75$ bit/sym. The degree distribution of C_1 is optimized to achieve the maximal rate R_0. With $2 - \varepsilon = 1.5$ bit/sym and $H(y_r|y_d) = 1.25$ bit/sym, the nested code C_2 has $R_b \geq 1.5$ bit/sym, $R_p \geq 1.25$ bit/sym.

Optimizing the degree distribution of LDPC code by (6) with $I_{q,pri} = 0.75$, $d_s = 16$, $R_0 = 0.742$ is acquired, and the optimized degree distribution v_{qd} is shown in Table II. With $d_b = 6$, $I_{yc} = 0.75$, the degree distribution of LDGM part of C_2 is optimized by (9), and $R_b = 1.5019$ is acquired. The optimized degree distribution v_{cd} of LDGM part at encoding side is represented in Table III.

TABLE II
THE DEGREE DISTRIBUTION OF LDPC CODE C_1 AT THE SOURCE

d	v_{qd}	d	v_{qd}	d	v_{qd}	d	v_{qd}	d	v_{qd}
2	0.2467	5	0.0154	8	0.0679	13	0.0027	21	0.0689
3	0.1768	5	0.0473	9	0.046	17	0.0067	24	0.0835
4	0.0479	6	0.0712	10	0.018	19	0.0412	27	0.0598

With $I_{c0} = 0.0472$ and $I_{c1} = 0.2028$, The EBP curve of LDGM part at the decoder-side is shown in the dashed curve of Fig.4. Then by (15), $R_p = 1.2696$ is acquired and the optimized degree distributions v_{pd}, v_{bd} of LDPC part at the decoder-side is shown in Table IV. The EBP curve of LDPC part is shown in the solid curve of Fig.4 with some gap. It can be seen that The EBP curve of LDPC part indeed lies below

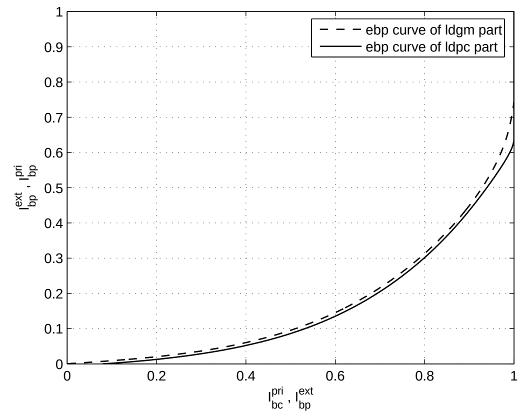

Fig. 4. The EBP curve of nested LDGM-LDPC codes in C_2 with gap at destination node

that of LDGM part and both of them match well, which assure that the iterative BP algorithm converge.

With optimized degree distribution of C_1 and C_2, the simulation of encoding and decoding processes is executed. The experiment result shows that the iteration count of BP decoding the LDGM part in C_2 is about 200, and LDPC decoding in destination is only about 150. However, when erasure probability ε decreases, which denotes the channel capacity increases, the iteration count will decrease. With Monte Carlo simulation, The BER performance at destination node for relay system in BEC under CF and DF is shown in Table V. Simulation shows that under CF the BER is about 10^5 in most blocks, and some blocks even recover source information correctly, which sees that CF is much better than DF ($R \leq 0.5$, by only optimizing the degree distribution of C_1 to realize DF). Besides, $R_0 = 0.472$ is close to CF theoretical limit 0.75.

Some remarks on the design of C_1 and C_2 are in order.

TABLE III
THE DEGREE DISTRIBUTION v_{cd} OF LDGM PART IN C_2

d	v_{cd}	d	v_{cd}	d	v_{cd}	d	v_{cd}	d	v_{cd}
1	0.002	6	0.026	11	0.0081	21	0.0033	37	0.0016
2	0.5987	7	0.0161	13	0.0074	24	0.0025	17	0.0013
3	0.1598	8	0.0126	15	0.0056	27	0.0021	19	0.0012
4	0.0175	9	0.0099	17	0.0047	30	0.0019	19	0.001
3	0.0408	10	0.0089	19	0.0043	33	0.0018		

TABLE IV
THE DEGREE DISTRIBUTION v_{bd},v_{pd} OF LDPC PART IN C_2

d	v_{bd}	d	v_{bd}	d	v_{bd}	d	v_{bd}	d	v_{pd}
1	0.0039	4	0.0173	7	0.1917	21	0.0947	2	0.6087
2	0.6505	7	0.0009	13	0.0303	24	0.0108	17	0.3913

TABLE V
BER PERFORMANCE AT DESTINATION: BEC CHANNEL; SINGLE-RELAY SYSTEM

Relay Protocol	Designed Rate	BER
CF	$R = 0.472$	1.357×10^{-5}
DF	$R = 0.49$	3.265×10^{-5}

Firstly, when the degree distribution of LDPC in C_1 and LDGM in C_2 is optimized, the degree of s-nodes and b-nodes should be chosen carefully. Here reasonable choice of d_s ranges from 16 to 20 and $d_b = 6$. Secondly, we should leave a uniform gap between the EXIT curves of LDPC codes in C_1 and between EBP curves of the LDGM and LDPC parts in C_2 to make iterative decoding converge with a reasonable number of iterations rather than getting stuck. Besides, in order not to cut down the designed rate, we need assure the gap $\triangle_{qs}$ and $\triangle_{pb}$ should not be larger than 0.01 in every iteration. E.g. this gap of the EBP curves of the nested LDGM-LDPC part in C_2 is designed with $\triangle_{pb}= 0.004$, which is shown in Fig.4. The decoding process of $\boldsymbol{y_r}$ will not get stuck and with the flipped position known in the destination, $\boldsymbol{y_r}$ could be decoded correctly.

Thirdly, there will inevitably be some incorrect decimation in the LDGM quantization process, which cause contradictions that must be corrected by flipping some bits in $\boldsymbol{\zeta}$. This $\boldsymbol{\zeta}$ must also be transmitted to the destination using a fraction of C_{rd}, so that it can perform decoding correctly. We found that the number of flipped positions is about 600, and R_p is 1.2696bit/sym, so we require $C_{rd} = R_p + 2 * H_2(600/2 * 10^5) = 1.8385$ bit/sym (here $H_2(p) = p\log_2 p + (1-p)\log_2(1-p)$). We could see that even with the flipped positions transmitted the required channel capacity C_{rd} of R-D link can still be lower than $H(y_r) = 1.5$ bit/sym. In our future work, we will study the lossy compression of $\boldsymbol{y_r}$, so that we can get lower C_{rd}.

It is also observed that the performance of the proposed practical CF scheme improves as the block length increases at the cost of larger memory consumption and coding delay.

VI. CONCLUSION

In this paper, a first practical CF scheme for a type of relay system based on nested LDGM-LDPC has been proposed, and methods for optimizing the degree distributions have been described. Simulation results show that the nested LDGM-LDPC codes can perform Slepian-Wolf compression of the relay's ternary received signals when the relay system is BEC. The performance of our scheme approaches the CF theoretical cut-set bound, while previous schemes are either limited to binary signals or suboptimal. Our work shows nested LDGM-LDPC codes for practical CF scheme is sufficient.

It is apparently straightforward to extend the proposed scheme to realize lossy compression, which would offer better performance achieved at a lower relay-destination channel capacity . The design will be considered in our future work. We will also try to optimize the gap between the BEP curves of the LDGM and LDPC parts of the nested code, so that its decoding can converge more quickly and reliably.

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